King Fahd University of Petroleum and Minerals
ICS Department

ICS-202 Data Structures

Assignment 1

First Semester 2021-22

Student ID									

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ICS 202 – Data Structures and Algorithms
First Semester 2021-22 (20211)
SOLUTION to Assignment # 1
Due on Saturday, 25 September 2021, 11.59pm

Q. 1: [4 marks] By choosing appropriate values for c and N, show that $f(n)$ is $O(n^2)$.

[Recall: $f(n)$ is $O(g(n))$ if there exist positive numbers c and N such that
$f(n) \le c \cdot g(n)$ for $n \ge N$]

$$f(n) = 4n^2 + 3\,n + 10$$

Solution:

Consider $g(n) = n^2$. Then $f(n)$ is $O(g(n))$ iff $f(n) \le c \cdot g(n)$ for $n \ge N$

Therefore, let $f(n) \le c \cdot g(n)$

$4n^2 + 3n + 10 \le c \cdot n^2$

$c \ge 4 + 3/n + 10/n^2$

For $N = 1$, $c \ge + 4 + 3/1 + 10/1^2$

Or $c \ge 17$.

Therefore for N = 1, $c \ge 17$, $f(n)$ is $O(g(n))$.

Q. 2: [6 marks] Consider the following functions in terms of n ($lg\ n$ is $\log_2 n$):

$$(4 / 3)^n, \quad n^3, \quad (lg\ n)^2, \quad n!, \quad n \lg n, \quad 1.$$

(a) Find the complexity class of each of these functions.
(b) Order these functions from the most efficient to the least efficient in terms of their complexity classes.

You may fill in the following table:

Solution:

Function	O(Complexity)	Order
1	$O(1)$	Most Efficient (Fastest)
$(lg\ n)^2$	$O(lg^2\ n)$	
$n \lg n$	$O(n \lg n)$	
n^3	$O(n^3)$	
$(4 / 3)^n$	$O((4 / 3)^n)$	
$n!$	$O(n!)$	Least Efficient (Slowest)

Question 3 (10 + 10 + 20 = 40 marks):
SOLUTION:

Q.3 (a)

$\#$ of iterations

$$= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (1)$$

$$= \sum_{i=0}^{n-1} (n-1-0+1) = \sum_{i=0}^{n-1} n$$

$$= (n-1-0+1)\cdot(n) = n^2 : \text{ANSWER}$$

Complexity = $O(n^2)$

(b)

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} (1) = (n-1-0+1)^3$$

$$= n^3.$$

Complexity = $O(n^3)$

(c)

$$\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} (3) = 3\sum_{i=0}^{n-2} (n-1-i-1+1)$$

$$= 3\sum_{i=0}^{n-2} (n-i-1)$$

$$= 3\sum n - 3\sum i - 3\sum 1$$

$$= 3n\,(n-2-0+1) - 3\frac{(n-2)(n-1)}{2} - 3(n-2-0+1)$$

$$= 3n(n-1) - \frac{3}{2}(n-1)(n-2) - 3(n-1)$$

$$= (n-1)\left[3n - \frac{3}{2}(n-2) - 3\right]$$

Complexity: $O(n^2)$

Question 4 (10 + 10 + 15 + 15 = 50 marks):

SOLUTION

Q.4 (a)

$$\#\text{ of iterations} = \sum_{i=1}^{n} \sum_{j=1}^{n} (1) = (n-1+1)^2 = n^2 \quad : \text{ANSWER}$$

$O(n^2)$

(b)

$$\sum_{i=1}^{n} \sum_{j=1}^{i} (1) = \sum_{i=1}^{n} (i-1+1) = \sum_{i=1}^{n} i$$

$$= \frac{n(n+1)}{2} \quad : \text{ANSWER}$$

$O(n^2)$

(c)–(d) are multiplicative loops.
For these (for the outer-loop).
$i = 1, 2, 4, 8, \ldots, 2^k$ [where $n = 2^k$]

Consider a variable 'r' where 'r' varies from 0 to k, i.e.

$i = 2^0, 2^1, 2^2, \ldots 2^r, \ldots, 2^k$.

Then when $i = 1, 2, 4, \ldots, 2^k \rightarrow r = 0, 1, \ldots, k$.

For (c)

$$\sum_{r=0}^{k} \sum_{j=1}^{n} (1) = \sum_{r=0}^{k} (n-1+1) = (k-0+1) \cdot n$$

$O(n \log n)$

$$= (k+1)n = (\log_2 n + 1) \cdot n \quad : \text{ANSWER}$$

For (d)

$$\sum_{r=0}^{k} \sum_{j=1}^{i} (1) = \sum_{r=0}^{k} (i) = \sum_{r=0}^{k} 2^r = \frac{2^{k+1}-1}{2-1}$$

END OF ASSIGNMENT