**7.2**

**Q8. What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?**

Note that there are 26! permutations of the letters, so the denominator in all of our answers is 26!. To find

the numerator, we have to count the number of ways that the given event can happen. Alternatively, in some cases we may be able to exploit symmetry.

**a) The first 13 letters of the permutation are in alphabetical order.**

There are 13! possible arrangements of the first 13 letters of the permutation, and in only one of these are they in alphabetical order. Therefore the answer is 1/13!.

**b) *a* is the first letter of the permutation and *z* is the last letter.**

Once these two conditions are met, there are 24! ways to choose the remaining letters for positions 2 through 25. Therefore the answer is 24!/26! = 1/650.

**c) *a* and *z* are next to each other in the permutation.**

In effect we are forming a permutation of 25 items—the letters b through y and the double letter combination az or za. There are 25! ways to permute these items, and for each of these permutations there are two choices as to whether a or z comes first. Thus there are 2 · 25! ways for form such a permutation, and therefore the answer is 2 · 25!/26! = 1/13.

**d) *a* and *b* are not next to each other in the permutation.**

By part (c), the probability that a and b are next to each other is 1/13. Therefore the probability that a and b are not next to each other is 12/13.

**e) *a* and *z* are separated by at least 23 letters in the permutation.**

There are six ways this can happen: az , za, xaz , xza, azx, and zax, where x stands for any letter other than a and z (but of course all the x’s are different in each permutation). In each of these there are 24! ways to permute the letters other than a and z , so there are 24! permutations of each type. This gives a total of 6 · 24! Permutations meeting the conditions, so the answer is (6 · 24!)/26! = 3/325.

**f ) *z* precedes both *a* and *b* in the permutation.**

Looking at the relative placements of z , a, and b, we see that one third of the time, z will come first. Therefore the answer is 1/3.

**Q14**

As instructed, we assume that births are independent and the probability of a birth in each day is 1/7. (This

is not exactly true; for example, doctors tend to schedule C-sections on weekdays.)

**a) What is the probability that two people chosen at random were born on the same day of the week?**

The probability that the second person has the same birth day-of-the-week as the first person (whatever

that was) is 1/7.

**b) What is the probability that in a group of *n* people chosen at random, there are at least two born on the same day of the week?**

We proceed as in Example 13. The probability that all the birth days-of-the-week are different is

Pn =

since each person after the first must have a different birth day-of-the-week from all the previous people in the group. Note that if n ³ 8, then pn = 0 since the seventh fraction is 0 (this also follows from the pigeonhole principle). The probability that at least two are born on the same day of the week is therefore 1 − pn .

**c) How many people chosen at random are needed to make the probability greater than 1*/*2 that there are at least two people born on the same day of the week?**

We compute 1−pn for n = 2, 3, . . . and find that the first time this exceeds 1/2 is when n = 4, so that is our answer. With four people, the probability that at least two will share a birth day-of-the-week is 223/343, or about 65%.

**Q24Find the probability that a randomly generated bit string of length 10 does not contain a 0 if bits are independent and if**

**a) a 0 bit and a 1 bit are equally likely.**

The probability that all bits are a 1 is = 1/1024. This is what is being asked for.

**b) the probability that a bit is a 1 is 0.6.**

This is the same as part (a), except that the probability of a 1 bit is 0.6 rather than 1/2. Thus the answer

is = 0.0060.

**c) the probability that the *i*th bit is a 1 is 1*/*2*i* for**

***i* = 1*,* 2*,* 3*, . . . ,* 10.**

We need to multiply the probabilities of each bit being a 1, so the answer is

Note that this is essentially 0.