**1.5 Nested Quantifiers**

**Q4. Let T (x, y) mean that student x likes cuisine y, where the domain for x consists of all students at your school and the domain for y consists of all cuisines. Express each of these statements by a simple English sentence**.

**a) ¬T (Abdallah Hussein, Japanese)**

Abdallah Hussein does not like Japanese cuisine.

**b) ∃xT (x, Korean) ∧ ∀xT (x, Mexican)**

There is some student at your school who likes Korean cuisine, and everyone at your school likes Mexican cuisine.

**c) ∃y(T (Monique Arsenault, y) ∨ T (Jay Johnson, y))**

There exists some cuisine that either Monique Arsenault or Jay Johnson likes

**d) ∀x∀z∃y((x = z)→¬(T (x, y) ∧ T (z, y)))**

For every pair of distinct students at your school, there is some cuisine that at least one them does not like.

**e) ∃x∃z∀y(T (x, y) ↔ T (z,y))**

There are two students at your school who like exactly the same set of cuisines

**f ) ∀x∀z∃y(T (x, y) ↔ T (z,y))**

For every pair of students at your school, there is some cuisine about which they have the same opinion (either they both like it or they both do not like it).

**Q14. Suppose the domain of the propositional function P(x, y) consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.**

**a) ∀x∀yP(x, y)**

P(1,1) ∧ P(1,2) ∧ P(1,3) ∧ P(2,1) ∧ P(2,2) ∧ P(2, 3) ∧ P(3, 1) ∧ P(3, 2) ∧ P(3, 3)

**b) ∃x∃yP(x, y)**

P(1, 1) ∨ P(1, 2) ∨ P(1, 3) ∨ P(2, 1) ∨ P(2, 2) ∨ P(2, 3) ∨ P(3,1)∨ P(3, 2) ∨ P(3, 3)

**c) ∃x∀yP(x, y)**

(P (1, 1) ∧ P(1, 2) ∧ P(1, 3)) ∨ (P (2 , 1) ∧ P(2, 2) ∧ P(2, 3)) ∨ (P (3, 1) ∧ P(3, 2) ∧ P(3, 3))

**d) ∀x∃yP(x, y)**

(P (1, 1) ∨ P(1, 2) ∨ P(1, 3)) ∧ (P (2, 1) ∨ P(2, 2) ∨ P(2, 3)) ∧ (P (3, 1) ∨ P(3, 2) ∨ P(3, 3)

**e) ∀y∃xP(x, y)**

(P (1, 1) ∨ P(2, 1) ∨ P(3, 1)) ∧ (P (1, 2) ∨ P(2, 2) ∨ P(3, 2)) ∧ (P (1, 3) ∨ P(2, 3) ∨ P(3, 3))

**Q16. Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).**

**a) ¬∃y∃xP(x, y)**

∀y∀x¬P(x, y)

**b) ¬∀x∃yP(x, y)**

∃x∀y ¬P(x, y)

**c) ¬∃y(Q(y) ∧ ∀x¬R(x, y))**

∀y(¬Q(y) ∨ ∃xR(x, y))

**d) ¬∃y(∃xR(x, y) ∨ ∀xS(x, y))**

∀y(∀x¬R(x, y) ∧ ∃x¬S(x, y))

**e) ¬∃y(∀x∃zT (x, y, z) ∨ ∃x∀zU(x, y, z))**

∀y(∃x∀z ¬T(x, y, z) ∧ ∀x∃z ¬U(x, y, z))