**6.2**

**Q12**

**a) Show that if seven integers are selected from the first**

**10 positive integers, there must be at least two pairs**

**of these integers with the sum 11.**

We can group the first ten positive integers into five subsets of two integers each, each subset adding

up to 11: {1, 10}, {2, 9}, {3, 8}, {4, 7}, and {5, 6}. If we select seven integers from this set, then by the

pigeonhole principle at least two of them come from the same subset. Furthermore, if we forget about these

two in the same group, then there are five more integers and four groups; again the pigeonhole principle

guarantees two integers in the same group. This gives us two pairs of integers, each pair from the same group.

In each case these two integers have a sum of 11, as desired.

**b) Is the conclusion in part (a) true if six integers are**

**selected rather than seven?**

No. The set {1, 2, 3, 4, 5, 6} has only 5 and 6 from the same group, so the only pair with sum 11 is 5

and 6.

**Q21. Show that in a group of 10 people (where any two people are either friends or enemies), there are either three mutual friends or four mutual enemies, and there are either three mutual enemies or four mutual friends.**

**Q22. Use Exercise 21 to show that among any group of 20 people (where any two people are either friends or enemies), there are either four mutual friends or four mutual**

**enemies.**

Let A be one of the people. She must have either 10 friends or 10 enemies, since if there were 9 or fewer of

each, then that would account for at most 18 of the 19 other people. Without loss of generality assume that

A has 10 friends. By Exercise 27 there are either 4 mutual enemies among these 10 people, or 3 mutual

friends. In the former case we have our desired set of 4 mutual enemies; in the latter case, these 3 people

together with A form the desired set of 4 mutual friends.

**Q25. Show that there are at least six people in California (population: 37 million) with the same three initials who were born on the same day of the year (but not necessarily in the same year). Assume that everyone has three initials.**

There are 6,432,816 possibilities for the three initials and a birthday. So, by the generalized pigeonhole principle, there are at least 37,000,000/6,432,816 = 6 people who share the same initials and birthday