**1.6 Rules of Inference**

**Q2. What rule of inference is used in each of these arguments?**

**a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major**.

Addition

**b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.**

Simplification

**c) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed**.

Modus ponens

**d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.**

Modus tollens

**e) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.**

Hypothetical syllogism

**Q8. For each of these arguments, explain which rules of inference are used for each step.**

**a) ∧Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.**

Let c(x) be ∧x is in this class.

let r(x) be ∧x owns a red convertible

let t(x) be ∧x has gotten a speeding ticket.

They give us these premises c(Linda), r(Linda), ∀x(r(x) → t(x)), and we want to conclude

∃x(c(x) ∧ t(x)).

**Step Reason**

[1] ∀x(r(x) → t(x)) Hypothesis

[2] r(Linda) → t(Linda) Universal instantiation using [1]

[3] r(Linda) Hypothesis

[4] t(Linda) Modus ponens using [2] and [3]

[5] c(Linda) Hypothesis

[6] c(Linda) ∧ t(Linda) Conjunction using [4] and [5]

[7] ∃x(c(x) ∧ t(x)) Existential generalization using [6]

**b) ∧Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year. ∧**

Let r(x) be ∧r is one of the five roommates listed.

let d(x) be ∧x has taken a course in discrete mathematics.

let a(x) be ∧x can take a course in algorithms

We are given premises ∀x(r(x) → d(x)) and ∀x(d(x) → a(x)), and we want to conclude

∀x(r(x) → a(x)).

(y represents an arbitrary person).

**Step Reason**

[1] ∀x(r(x) → d(x)) Hypothesis

[2] r(y) → d(y) Universal instantiation using [1]

[3] ∀x(d(x) → a(x)) Hypothesis

[4] d(y) → a(y) Universal instantiation using [3]

[5] r(y) → a(y) Hypothetical syllogism using [2] and [4]

[6] ∀x(r(x) → a(x)) Universal generalization using [5]

**c) ∧All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners. ∧**

Let s(x) be ∧x is a movie produced by Sayles

let c(x) be ∧x is a movie about coal miners

let w(x) be ∧movie x is wonderful

It was given premises ∀x(s(x) → w(x)) and ∃x(s(x) ∧ c(x)), and we want to conclude ∃x(c(x) ∧ w(x)).

(y represents an unspecified particular movie).

**Step Reason**

[1] ∃x(s(x) ∧ c(x)) Hypothesis

[2] s(y) ∧ c(y) Existential instantiation using [1]

[3] s(y) Simplification using [2]

[4] ∀x(s(x) → w(x)) Hypothesis

[5] s(y) → w(y) Universal instantiation using [4]

[6] w(y) Modus ponens using [3] and [5]

[7] c(y) Simplification using [2]

[8] w(y) ∧ c(y) Conjunction using [6] and [7]

[9] ∃x(c(x) ∧ w(x)) Existential generalization using [8]

**d) ∧There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre. ∧**

Let c(x) be ∧x is in this class.

let f(x) be ∧x has been to France.

and let l(x) be ∧x has visited the Louvre.

It was given ∃x(c(x) ∧ f(x)), ∀x(f(x) → l(x)), and we want to conclude ∃x(c(x) ∧ (x)).

(y represents an unspecified particular person).

**Step Reason**

[1] ∃x(c(x) ∧ f(x)) Hypothesis

[2] c(y) ∧ f(y) Existential instantiation using [1]

[3] f(y) Simplification using [2]

[4] c(y) Simplification using [2]

[5] ∀x(f(x) → l(x)) Hypothesis

[6] f(y) → l(y) Universal instantiation using [5]

[7] l(y) Modus ponens using [3] and [6]

[8] c(y) ∧ l(y) Conjunction using [4] and [7]

[9] ∃x(c(x) ∧ l(x)) Existential generalization using [8]

**Q16. Identify the error or errors in this argument that supposedly shows that if ∀x(P(x) ∨ Q(x)) is true then ∀xP(x) ∨ ∀xQ(x) is true.**

**1. ∀x(P(x) ∨ Q(x)) Premise**

**2. P(c) ∨ Q(c) Universal instantiation from (1)**

**3. P(c) Simplification from (2)**

**4. ∀xP(x) Universal generalization from (3)**

**5. Q(c) Simplification from (2)**

**6. ∀xQ(x) Universal generalization from (5)**

**7. ∀x(P(x) ∨ ∀xQ(x)) Conjunction from (4) and (6)**

Steps [3] and [5] are incorrect.

We can apply simplification to conjunctions, but we cannot apply it to disjunctions.