**7.1**

**Q18. What is the probability that a positive integer not exceeding 100 selected at random is divisible by 3?**

There are [100/3] = 33 multiples of 3 among the integers from 1 to 100 (inclusive)

so the answer is 33/100 = 0.33 .

**Q24. Suppose that 100 people enter a contest and that different winners are selected at random for first, second, and third prizes. What is the probability that Kumar, Janice, and Pedro each win a prize if each has entered the contest?**

The number of ways for the drawing to turn out is 100 · 99 · 98.

The number of ways of ways for the drawing to cause Kumar, Janice, and Pedro each to win a prize is 3 · 2 · 1 (three ways for one of these to be picked to win first prize, two ways for one of the others to win second prize, one way for the third to win third prize).

Therefore the probability we seek is (3 · 2 · 1)/(100 · 99 · 98) = 1/161700.

**Q30. Two events E1 and E2 are called independent if p(E1 ∩ E2) = p(E1)p(E2). For each of the following pairs of events, which are subsets of the set of all possible**

**outcomes when a coin is tossed three times, determine whether or not they are independent.**

**a) E1: tails comes up with the coin is tossed the first time; E2: heads comes up when the coin is**

**tossed the second time**

Intuitively, these should be independent, since the first event seems to have no influence on the second.

In fact we can compute as follows. First p(E1) = 1/2 and p(E2) = 1/2 by the symmetry of coin tossing.

Furthermore, E1**∩**E2 is the event that the first two coins come up tails and heads, respectively.

Since there are four equally likely outcomes for the first two coins (HH, HT , TH, and TT ), p(E1 **∩** E2) = 1/4. Therefore

p(E1 **∩** E2) = 1/4 = (1/2) · (1/2) = p(E1)p(E2), so the events are indeed independent.

**b) E1: the first coin comes up tails; E2: two, and not three, heads come up in a row.**

Again p(E1) = 1/2. For E2 , note that there are 8 equally likely outcomes for the three coins, and in

2 of these cases E2 occurs (namely HHT and THH); therefore p(E2) = 2/8 = 1/4. Thus p(E1)p(E2) =

(1/2) · (1/4) = 1/8. Now E1 **∩** E2 is the event that the first coin comes up tails, and two but not three heads

come up in a row. This occurs precisely when the outcome is THH, so the probability is 1/8. This is the

same as p(E1)p(E2), so the events are independent.

**c) E1: the second coin comes up tails; E2: two, and not three, heads come up in a row.**

As in part (b), p(E1) = 1/2 and p(E2) = 1/4. This time p(E1 **∩** E2) = 0, since there is no way to get

two heads in a row if the second coin comes up tails. Since p(E1)p(E2) '= p(E1 **∩** E2), the events are not

independent.