**5.2 Strong Induction and Well-Ordering**

**Q4. Let P(n) be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The 342 5 / Induction and Recursion parts of this exercise outline a strong induction proof that P(n) is true for n ≥ 18.**

**a) Show statements P(18), P(19), P(20), and P(21) are true, completing the basis step of the proof.**

P(18) is true, because we can form 18 cents of postage with one 4-cent stamp and two 7-cent stamps.

P(19) is true, because we can form 19 cents of postage with three 4-cent stamps and one 7-cent stamp.

P(20) is true, because we can form 20 cents of postage with five 4-cent stamps.

P(21) is true, because we can form 20 cents of postage with three 7-cent stamps.

**b) What is the inductive hypothesis of the proof?**

The inductive hypothesis is that

using just 4-cent and 7-cent stamps we can form j cents postage for all j with 18 £ j £k , where we assume that k ³ 21.

**c) What do you need to prove in the inductive step?**

we must show that we can form k + 1 cents postage using just 4-cent and 7-cent stamps.

**d) Complete the inductive step for k ≥ 21.**

We want to form k + 1 cents of postage.

Since k ³ 21, we know that P(k − 3) is true, that is what we can form k−3 cents of postage. Put one more 4-cent, then we have formed k+1 cents of postage.

**e) Explain why these steps show that this statement is**

We completed and the inductive prove, so the statement is true for every integer n greater than or equal to 18.

**Q8. Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers =1,**

**=2, =4, and so on.**

The basis step is to note that 1 = . Notice for subsequent steps that

2 =

3 = +

4 =

5 = + , and so on

this is the representation of a number in binary form.

Assume that every positive integer up to k can be written as a sum of distinct powers of 2. We must show that k + 1 can be written as a sum of distinct powers of 2.

If k + 1 is odd, then k is even, so was not part of the sum for k .

Therefore the sum for k+1 is the same as the sum for k with the extra term added.

If k +1 is even, then (k +1)/2 is a positive integer, so (k + 1)/2 can be written as a sum of distinct powers of 2 (by the inductive hypothesis). Increasing each exponent by 1 doubles the value and gives us the sum for k + 1.