**5.3 Recursive Definitions and Structural Induction**

**Q4. Give a recursive definition of the sequence {}, n = 1, 2, 3, . . . if**

**a) = 6n.**

+ 1 = + 6 for n ≥ 1and = 6

**b) = 2n + 1.**

+1 = + 2 for n ≥ 1 and = 3

**c) = 10n.**

+1 = 10 for n ≥ 1 and = 10

**d) = 5.**

+1 = for n ≥ 1 and = 5

**Q18. Let S be the subset of the set of ordered pairs of integers**

**defined recursively by**

**Basis step:**

**(0, 0) ∈ S.**

**Recursive step:**

**If (a, b) ∈ S, then (a, b + 1) ∈ S, (a + 1, b + 1) ∈ S, and (a + 2, b + 1) ∈ S.**

**a) List the elements of S produced by the first four applications**

**of the recursive definition.**

(0, 1), (1, 1), (2, 1);

(0, 2), (1, 2), (2, 2), (3, 2), (4, 2);

(0, 3), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3);

(0, 4), (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (7, 4), (8, 4)

**b) Use strong induction on the number of applications of**

**the recursive step of the definition to show that a ≤ 2b**

**whenever (a, b) ∈ S.**

Let P(n) be the statement a ≤ 2b whenever (a, b) ∈ S is obtained by n applications of the recursive step.

Basis step:

P(0) is true, because the only element of S obtained with no applications of the recursive step is (0, 0), and 0 ≤ 2 · 0.

Inductive step:

Assume that a ≤ 2b whenever (a, b) ∈ S is obtained by k or fewer applications of the recursive step, Then consider k + 1 applications of the recursive step.

The final application of the recursive step to an element (a, b) must be applied to an element obtained with fewer applications of the recursive step.

we know that a ≤ 2b. Add 0 ≤ 2, 1 ≤ 2, and 2 ≤ 2, respectively, then we obtain a ≤ 2(b + 1), a + 1 ≤ 2(b + 1), and a + 2 ≤ 2(b + 1).

**c) Use structural induction to show that a ≤ 2b whenever**

**(a, b) ∈ S.**

This holds for the basis step, because 0 ≤ 0. If this holds for (a, b), then it also holds for the elements obtained from (a, b) in the recursive step, because adding 0 ≤ 2, 1 ≤ 2, and 2 ≤ 2, respectively, to a ≤ 2b yields a ≤ 2(b + 1), a + 1 ≤ 2(b + 1), and a + 2 ≤ 2(b + 1).

**Q25. Give a recursive definition of the reversal of a string Then write a string w of length n + 1 as xy, where x is a string of length n, and express the reversal of w in terms ofand y**

and

for x Î å and y Î å\* .