King Fahd University of Petroleum and Minerals
Information and Computer Science Department

ICS 253: Discrete Structures I

Homework Assignment #4
(Due Wednesday August 16, 2017 at midnight)

I. Solve and submit the solutions to Questions 1-10. In all questions below, show your intermediate work. Make sure to attempt these questions without using a calculator.

1) (7 points)How many subsets of a set with 100 elements have more than one element?

It is the total # of subsets minus the # of subsets with 0 or 1 elements

$$= 2^{100} - \left(\binom{100}{0} + \binom{100}{1} \right)$$

$$= 2^{100} - (1 + 100)$$

$$= 2^{100} - 101$$

<7, 5, 3, 1>

2) (8 points) How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15} to guarantee that at least one pair of these numbers add up to 16?

Since we have 5 pairs (1, 15), (3, 13), (5, 11) and (7, 9), by the pigeonhole principle, choosing 5 #'s $= \frac{8}{2}+1$ guarantees that we choose 1 pair.

<8, 6, 4, 2>

3) (10 points) A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

i. How many balls must she select to be sure of having at least three balls of the same color?

5 balls (r b r b $\underset{\text{whatever}}{\underline{x}}$)

<5, 3, 1>

ii. How many balls must she select to be sure of having at least three blue balls?

$10+3=13$ (In the worst scenario, she will choose the 1st 10 red balls before choosing blue balls.)

<5, 3, 1>

4) (10 points) Seven students and nine faculty members belong to the mathematics department at a school.

a. How many ways are there to select a committee of five members of the department if at least one student must be on the committee?

<5, 3, 1>

Let $c(i,j)$ denote that there are i students & j faculty members in the committee. Hence, the answer is number(1,4) + number(2,3) + number(3,2) + number(4,1) + number(5,0)

$$= \binom{7}{1}\binom{9}{4} + \binom{7}{2}\binom{9}{3} + \binom{7}{3}\binom{9}{2} + \binom{7}{4}\binom{9}{1} + \binom{7}{5}\binom{9}{0}.$$

b. How many ways are there to select a committee of five members of the department if at least one student and at least one faculty member must be on the committee?

<5, 3, 1>

$$\binom{7}{1}\binom{9}{4} + \binom{7}{2}\binom{9}{3} + \binom{7}{3}\binom{9}{2} + \binom{7}{4}\binom{9}{1}.$$

5) (10 points) Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?

number(4,2) + number(5,1) + number(6,0)

$$= \binom{10}{2}\binom{15}{4} + \binom{10}{1}\binom{15}{5} + \binom{10}{0}\binom{15}{6}$$

<10, 8, 6, 2>

6) (10 points) Give a formula for the coefficient of x^k in the expansion of $(x + 1/x)^{100}$, where k is an integer.

$$\left(x+\frac{1}{x}\right)^{100} = \sum_{j=0}^{100} \binom{100}{j} x^j \left(\frac{1}{x}\right)^{100-j}$$

$$= \sum_{j=0}^{100} \binom{100}{j} \frac{x^j \cdot x^j}{x^{100}}$$

$$= \sum_{j=0}^{100} \binom{100}{j} x^{2j-100}$$

$$\boxed{\begin{aligned} k &= 2j-100 \\ 2j &= k+100 \\ j &= \frac{k+100}{2} \end{aligned}}$$

$$\therefore \text{Coefficient of } x^k = \begin{cases} 0 & k \text{ is odd} \\ & -100 \le k \le 100 \\ \binom{100}{\frac{k+100}{2}} & k \text{ is even} \\ & \& -100 \le k \le 100 \end{cases}$$

<10, 8, 6, 2>

7) (5 points) How many ways are there to seat six people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table?

In total, we have 6! ways to seat six people around a circular table, where for each seating, there are 5 other equivalent seatings, for a total of 6 seatings. Hence, the answer is

$\langle 5, 3, 1 \rangle$ $$\frac{6!}{6} = 5!$$

8) (10 points) What is the probability that a five-card poker hand contains the ace of hearts?

$|S| = \binom{52}{5}$.

Since there is only one way to choose the ace of hearts $\left[\binom{1}{1}\right]$ & since choosing the other four cards is done where the order is not important $\left[\binom{51}{4}\right]$

$\therefore$ the answer is $$\frac{\binom{1}{1}\binom{51}{4}}{\binom{52}{5}} = \frac{5}{52}.$$

$\langle 10, 8, 6, 4, 2 \rangle$

9) (10 points) What is the probability that a five-card poker hand contains the two of diamonds and the three of spades?

$$\frac{\binom{1}{1}\binom{1}{1}\binom{50}{3}}{\binom{52}{5}}$$

$\langle 10, 8, 6, 4, 2 \rangle$

10) (10 points) What is the probability that a five-card poker hand contains a flush, that is, five cards of the same suit?

Let n_s be the number of ways to choose a suit
Let n_f be the " " " " choose 5 cards from 1 suit. Then,
the answer is $\frac{n_s \cdot n_f}{\binom{52}{5}} = \frac{\binom{4}{1}\binom{13}{5}}{\binom{52}{5}}$

$\langle 10, 8, 6, 4, 2 \rangle$