King Fahd University of Petroleum and Minerals
Information and Computer Science Department

ICS 253: Discrete Structures I

Homework Assignment #2
(Due Monday July 24, 2017 at midnight)

I. Solve and submit the solutions to Questions 1-8. In all questions below, show your intermediate work. Make sure to attempt these questions without using a calculator.

1) (10 points) Show that $[(p \to q) \land (q \to r)] \to (p \to r)$ is a tautology without using a truth table.

$$((p \to q) \land (q \to r)) \to (p \to r)$$

$$\Leftrightarrow ((\neg p \lor q) \land (\neg q \lor r)) \to (p \to r)$$

$$\Leftrightarrow \neg((\neg p \lor q) \land (\neg q \lor r)) \lor (\neg p \lor r)$$

$$\Leftrightarrow ((p \land \neg q) \lor (q \land \neg r) \lor \neg p \lor r)$$

$$\Leftrightarrow ((p \land \neg q) \lor \neg p \lor (q \land \neg r) \lor r)$$

$$\Leftrightarrow ((p \lor \neg p) \land (\neg p \lor \neg q)) \lor [(q \lor \neg r) \land (\neg r \lor r)]$$

$$\Leftrightarrow T \land (\neg p \lor \neg q) \lor [(q \lor \neg r) \land T]$$

$$\Leftrightarrow \neg p \lor \neg q \lor q \lor \neg r$$

$$\Leftrightarrow \neg p \lor \neg r \lor T$$

$$\Leftrightarrow T$$

$$\langle 10, 9, 7, 4, 1, 0 \rangle$$

2) (10 points) Suppose that the domain of the propositional function *P(x)* consists of −5, −3, −1, 1, 3, and 5. Express the following statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

1. $\forall x\big((x \neq 1) \rightarrow P(x)\big)$
2. $\exists x\big(\neg P(x)\big) \wedge \forall x\big((x < 0) \rightarrow P(x)\big)$

1. $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$

<5,0>

2. $[\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)] \wedge [P(-5) \wedge P(-3) \wedge P(-1)]$

<5,0>

3) (10 points) Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall x\, P(x) \leftrightarrow \forall x Q(x)$ are logically equivalent. Justify your answer.

<4,0> No. [Let $U = \{1, 2\}$, $P(1) = T$, $P(2) = F$, $Q(1) = F$ and $Q(2) = T$. <6,4,2>

Then $\forall x(P(x) \Leftrightarrow Q(x))$ is false

However $\forall x\, P(x) \Leftrightarrow \forall x Q(x)$ is true.]

4) (30 points) Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

<3,1> (for each answer)

1. Everybody can fool Fred.

   $\forall x\, F(x, Fred)$

2. Evelyn can fool everybody.

   $\forall y\, F(Evelyn, y)$

3. Everybody can fool somebody.

   $\forall x\, \exists y\, F(x, y)$

4. There is no one who can fool everybody.

   $\neg \exists x\, \forall y\, F(x, y) \Leftrightarrow \forall x\, \exists y\, \neg F(x, y)$

5. Everyone can be fooled by somebody.

   $\forall y\, \exists x\, F(x, y)$

6. No one can fool both Fred and Jerry.

   $\neg \exists x (F(x, Fred) \wedge F(x, Jerry)) \Leftrightarrow \forall x (\neg F(x, Fred) \vee \neg F(x, Jerry))$

7. Nancy can fool exactly two people.

   $\exists x\, \exists y\, ((x \neq y) \wedge F(Nancy, x) \wedge F(Nancy, y) \wedge \neg \exists z (F(Nancy, z) \wedge (z \neq x) \wedge (z \neq y)))$

8. There is exactly one person whom everybody can fool.

   $\exists y\, \forall x (F(x, y) \wedge \neg \exists z (\forall w (F(w, z)) \wedge (z \neq x)))$

9. No one can fool himself or herself.

   $\neg \exists x\, (F(x, x)) \Leftrightarrow \forall x\, (\neg F(x, x))$

10. There is someone who can fool exactly one person besides himself or herself.

   $\exists x\, \exists y\, ((x \neq y) \wedge F(x, x) \wedge F(x, y) \wedge \neg \exists z\, ((z \neq y) \wedge (z \neq x) \wedge F(x, z)))$

   [assuming he can fool himself also. An answer without this assumption is also correct.

5) (9 points) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

1. $\forall x \exists y (x = 1/y)$

   $x=0$ has no y s.t. $0 = \frac{1}{y}$ $\langle 3, 0 \rangle$

2. $\forall x \exists y (y^2 - x \not< 100)$

   There exists no y s.t. $y^2 - (-100) < 100$ (i.e. $y^2 < 0$) $\langle 3, 0 \rangle$

3. $\forall x \forall y (x^2 \neq y^3)$

   $x = 0$, $y = 0$.

   $\langle 3, 0 \rangle$

6) (6 points) Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain for the variables consists of

1. the positive real numbers.

   $\langle 2, 0 \rangle$ False. no x makes $x \leq 0$ true

2. the integers.

   $\langle 2, 0 \rangle$ True.

3. the nonzero real numbers.

   $\langle 2, 0 \rangle$ True.

7) (15 points) For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

1. "If I play hockey, then I am sore the next day." "I use the whirlpool if I am sore." "I did not use the whirlpool."

p q r

1. $p \to q$
2. $q \to r$ <2,0>
3. $\neg r$

4. 1 & 2 $\to$ $p \to r$ (Hypothetical Syllogism). <3,0>
5. 2 & 3 $\to$ $\neg q$ (Modus Tollens) <3,0>
6. 5 & 1 $\to$ $\neg p$ (Modus Tollens) <3,0>

2. "If I work, it is either sunny or partly sunny." "I worked last Monday or I worked last Friday." "It was not sunny on Tuesday." "It was not partly sunny on Friday."

Let $W(x) \equiv$ I worked on day x.
$S(x) \equiv$ Day x is sunny
$P(x) \equiv$ Day x is partly sunny

We have
1. $W(x) \to S(x) \vee P(x)$.
2. $W(\text{Monday}) \vee W(\text{Friday})$
3. $\neg S(\text{Tuesday})$
4. $\neg P(\text{Friday})$

<2,0>

$1 \leftrightarrow \neg W(x) \vee S(x) \vee P(x) \Leftrightarrow \neg P(x) \to S(x) \vee \neg W(x)$

5. $\neg P(x) \to S(x) \vee \neg W(x)$

Now, using 4 & 5, we conclude
$S(\text{Friday}) \vee \neg W(\text{Friday})$. <2,0>

8) (10 points) Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

Solution is available from Major 1 Solution last question. (Blackboard → Assesments & Tests)

$\langle 10, 9, 7, 4, 0 \rangle$