King Fahd University of Petroleum and Minerals
Information and Computer Science Department

ICS 253: Discrete Structures I

Homework Assignment #1
(Due Friday July 6, 2018 at midnight)

I. Solve and submit the solutions to Questions 1-9. In all questions below, show your intermediate work. Make sure to attempt these questions without using a calculator.

1) (10 points) Let p, q and r be the propositions "Bats are blind", "Camels store fat in their humps" and "Lions eat grass" respectively. Express each of these compound propositions as an English sentence.

   a) (3 points) $p \vee \neg r$

   Bats are blind or lions do not eat grass.

   b) (3 points) $q \rightarrow \neg p$

   If camels store fat in their humps then bats are not blind.

   c) (4 points) $(p \wedge q) \leftrightarrow \neg r$

   To say that bats are blind and camels store fat in their humps is equivalent to say that lions do not eat grass.

2) (6 points) For each of these sentences, state what the sentence means if the logical connective *or* is an inclusive *or* (that is, a disjunction) versus an exclusive *or*. Which of these meanings of *or* do you think is intended?

   a) The door was open or closed.

   Exclusive or. Both cannot happen at the same time.

   b) If your mother or father had high blood pressure, there is a good chance you might also.

   Inclusive or. No conflict between the two happenning.

   c) Our team will qualify if we win next game or bring it to a draw.

   Exclusive or. Both cannot happen at the same time.

3) (15 points) Let p, q, and r be the propositions

p : You can fly a plane.

q : You can drive a truck.

r : You can win a bike race.

Write these propositions using p, q, and r and logical connectives (including negations).

a) (3 points) You can't win a bike race unless you can fly a plane.

$\neg p \to \neg r$

b) (3 points) If you can fly a plane, then you can drive a truck, and vice versa.

$p \leftrightarrow q$

c) (3 points) For you to win a bike race, it is necessary to drive a truck.

$r \to q$

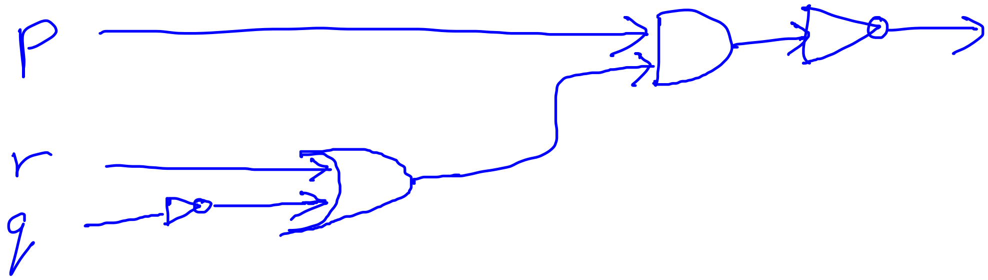
d) (3 points) You can drive a truck when you can fly a plane.

$p \to q$

e) (3 points) You cannot fly a plane, nevertheless you can drive a truck.

$\neg p \wedge q$

4) (5 points) Construct a combinatorial circuit using inverters, OR gates, and AND gates that produces the output $\neg((\neg q \vee r) \wedge p)$



5) (10 points) Tamer, Mohammad and Jihad belong to Everest Club. Every club member is either a skier or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Mohammad dislikes whatever Tamer likes and likes whatever Tamer dislikes. Tamer likes rain and snow. Is there a member of the Everest Club who is a mountain climber but not a skier? Clearly justify your answer.

Let $M(x) \equiv x$ is a mountain climber.

$S(x) \equiv x$ is a skier

$R(x) \equiv x$ likes rain

$N(x) \equiv x$ likes snow

where the domain of x is club members. Then,

1. $\forall x(M(x) \vee S(x))$
2. $\neg \exists x(M(x) \wedge R(x))$

$\Leftrightarrow \forall x(\neg M(x) \vee \neg R(x))$

$\Leftrightarrow \forall x(M(x) \rightarrow \neg R(x))$

3. $\forall x(S(x) \rightarrow N(x))$
4. $R(Mohammad) \leftrightarrow \neg R(Tamer)$
5. $N(Mohammad) \leftrightarrow \neg N(Tamer)$
6. $R(Tamer)$
7. $N(Tamer)$

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7 & 5 $\rightarrow \neg N(Mohammad)$ — 8

8 & 3 $\rightarrow \neg S(Mohammad)$ — 9

9 & 1 $\rightarrow M(Mohammad)$ — 10

9 & 10 $\rightarrow \neg S(Mohammad) \wedge M(Mohammad)$

$\therefore$ The answer is Yes. Mohammad.

6) (10 points) Show that $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology without using a truth table.

$\Leftrightarrow [(\neg p \vee q) \wedge (\neg q \vee r)] \rightarrow (\neg p \vee r)$

$\Leftrightarrow \neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee \neg p \vee r$

$\Leftrightarrow (p \wedge \neg q) \vee (q \wedge \neg r) \vee \neg p \vee r$

$\Leftrightarrow [(p \wedge \neg q) \vee \neg p] \vee [(q \wedge \neg r) \vee r]$

$\Leftrightarrow \{(p \vee \neg p) \wedge (\neg q \vee \neg p)\} \vee [(q \vee r) \wedge (\neg r \vee r)]$

$\Leftrightarrow \{T \wedge (\neg q \vee \neg p)\} \vee [(q \vee r) \wedge T]$

$\Leftrightarrow \neg q \vee \neg p \vee q \vee r$

$\Leftrightarrow (\neg q \vee q) \vee \neg p \vee r$

$\Leftrightarrow T \vee (\neg p \vee r)$

$\Leftrightarrow T$.

7) (10 points) Determine the truth value of each of these statements if the domain consists of all integers. Briefly prove each answer.

a) $\exists x(x^3 = -1)$

True. $x=-1$.

b) $\exists x(x^4 < x^2)$

False. $x^4 \geq x^2 \ \forall x \in \mathbb{Z}$.

c) $\forall x((-x)^2 = x^2)$

True. $(-x)^2 = (-1)^2 x^2 = x^2$.

d) $\forall x(2x > x)$

False. $2(-1) = -2 \not> -1$

8) (14 points) Suppose the domain of the propositional function *P(x, y)* consists of pairs *x* and *y*, where *x* is 1, 2, or 3 and *y* is 1 or 3. Write out these propositions using disjunctions and conjunctions.

a) $\exists x \forall y P(x,y)$

$(P(1,1) \wedge P(1,3)) \vee (P(2,1) \wedge P(2,3)) \vee (P(3,1) \wedge P(3,3))$

b) $\forall y \neg \exists x P(x,y)$

$\neg P(1,1) \wedge \neg P(1,3) \wedge \neg P(2,1) \wedge \neg P(2,3) \wedge \neg P(3,1) \wedge \neg P(3,3)$

c) $\forall x \exists y P(x,y)$

$(P(1,1) \vee P(1,3)) \wedge (P(2,1) \vee P(2,3)) \wedge (P(3,1) \vee P(3,3))$

9) (30 points) Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred.

$\forall x\, F(x, \text{Fred})$.

b) Evelyn can fool everybody.

$\forall y\, F(\text{Evelyn}, y)$

c) Everybody can fool somebody.

$\forall x\, \exists y\, F(x, y)$

d) There is no one who can fool everybody.

$\neg \exists x\, \forall y\, F(x, y) \Leftrightarrow \forall x\, \exists y\, \neg F(x, y)$

e) Everyone can be fooled by somebody.

$\forall y\, \exists x\, F(x, y)$

f) No one can fool both Fred and Jerry.

$\neg \exists x\, (F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$

g) Nancy can fool exactly two people.

$$\exists x\, \exists y\, ((x \neq y) \wedge F(\text{Nancy}, x) \wedge F(\text{Nancy}, y) \wedge \neg \exists z\, (F(\text{Nancy}, z) \wedge (z \neq x) \wedge (z \neq y)))$$

h) There is exactly one person whom everybody can fool.

$$\exists y\, \forall x\, (F(x, y) \wedge \neg \exists z\, (\forall w\, F(w, z) \wedge (z \neq y)))$$

$$\Leftrightarrow \exists y\, \forall x\, (F(x, y) \wedge \forall z\, (\forall w\, F(w, z) \rightarrow (z = y)))$$

i) No one can fool himself or herself.

$\neg \exists x\, F(x, x)$

j) There is someone who can fool exactly one person besides himself or herself.

$$\exists x\, \exists y\, (F(x, y) \wedge (x \neq y) \wedge \neg \exists z\, (F(x, z) \wedge (z \neq y) \wedge (z \neq x)))$$

$$\exists x\, \exists y\, (F(x, y) \wedge (x \neq y) \wedge \forall z\, ([F(x, z) \wedge (z \neq x)] \rightarrow z = y))$$