King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 253-01: Discrete Structures I
Summer 2012-2013
Quiz#2, Tuesday June 18, 2013.

Name:

ID#:

1. (10 points) Let F(x, y) be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.
   a. (2 points) Everybody can fool somebody.

$\forall x \exists y \, F(x,y)$

   b. (2 points) Everyone can be fooled by somebody.

$\exists x \forall y \, F(x,y)$

   c. (3 points) No one can fool both Fred and Jerry.

$\neg \exists x \, (F(x, Fred) \wedge F(x, Jerry))$
$\Leftrightarrow \forall x \, \neg F(x, Fred) \vee \neg F(x, Jerry)$

   d. (3 points) There is exactly one person whom everybody can fool.

$\exists x \forall y \, (F(y,x) \wedge \neg \exists z \, ((z \neq x) \wedge F(y,z)))$
$\Leftrightarrow \exists x \forall y (F(y,x) \wedge \forall z \, ((z = x) \vee \neg F(y,z)))$
$\Leftrightarrow \exists x \forall y \forall z \, (F(y,x) \wedge (F(y,z) \rightarrow (z = x)))$

2. (10 points) For each of these arguments determine whether the argument is correct or incorrect and explain why.

   a. (3 points) All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.

   $p(x) \equiv x$ is a student in this class $q(x) \equiv x$ understands logic

   1. $\forall x\,(p(x) \to q(x))$ 2. $p(\text{Xavier})$

   $\therefore\ q(\text{Xavier})$. True (Universal Instantiation)

   b. (2 points) Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.

   $p(x) \equiv x$ is computer science major $q(x) \equiv x$ takes discrete maths.

   1. $\forall x(p(x) \to q(x))$ 2. $q(\text{Natasha})$

   $\therefore\ p(\text{Natasha})$ *WRONG* $q(x) \not\to p(x)$.

   c. (2 points) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

   $p(x) \equiv x$ is a parrot. $q(x)$: x likes fruit.

   1. $\forall x\,(p(x) \to q(x))$ 2. $\neg p(\text{My Pet Bird})$

   $\therefore\ \neg q(\text{My Pet Bird})$ *WRONG* $\neg p(x) \not\to \neg q(x)$.

   d. (3 points) Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

   $p(x)$: x eats granola every day $q(x)$: x is healthy

   1. $\forall x\,(p(x) \to q(x))$ 2. $\neg q(\text{Linda})$

   $\therefore\ \neg p(\text{Linda})$. True Modus Tollens & Universal Instantiation.