**King Fahd University of Petroleum and Minerals**

**Information and Computer Science Department**

**ICS 253: Discrete Structures I**

**Homework Assignment #2**

**(Due Thursday June 20, 2013 at midnight)**

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1. (10 points) Let Q(x) be the statement “x + 1 > 2x.” If the domain consists of all integers, what are these truth values?
   1. Q(0) **T**
   2. Q(−1) **T**

2 points for the first 3, and 1 point for the last four.

* 1. Q(1) **F**
  2. ∃xQ(x) **T**
  3. ∀xQ(x) **F**
  4. ∃x¬Q(x) **T**
  5. ∀x¬Q(x) **F**

1. (10 points) Suppose that the domain of the propositional function P(x) consists of −5, −3, −1, 1, 3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

2 points

3 points

5 points (binary)

* 1. ∀x((x ≠ 1) → P(x))
  2. ∃x((x ≥ 0) ∧ P(x))
  3. ∃x(¬P(x)) ∧ ∀x((x < 0) → P(x))

1. (6 points) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

2 points each (binary)

* 1. ∀x(x2 ≠ x) **any answer other than 0 or 1.**
  2. ∀x(x2 ≠ 2)
  3. ∀x(|x| > 0) **0**

1. (6 points) Determine whether ∀x(P(x) ↔ Q(x)) and ∀x P(x) ↔ ∀xQ(x) are logically equivalent. Justify your answer.

**No they are not. The reason is that ∀x P(*x*) could be false because of only one element *x*1 where P(*x*1) is F and ∀xQ(x) could be false because of only one element *x*2 where Q(*x*2) is False, where (*x*1 ≠*x*2). However, the RHS is still true, but the LHS is false because P(*x*1) is not equivalent to Q(*x*1).** 6 points (binary)

1. (12 points) Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.
   1. ∀x(A → P(x)) ≡ A → ∀xP(x)

**If A is False, then clearly, both sides are true. If A is True, then if there is an x such that P(x) is false, then both sides are clearly false. Otherwise, if P(x) is true for all x, then both sides are True.** 3 points + 3 points (for each case)

* 1. ∃x(A → P(x)) ≡ A → ∃xP(x)

**If A is False, then clearly both sides are True. If A is true, then if P(x) is true for some value x, then, both sides are True. If P(x) is false for all x, then both sides are False.** 3 points + 3 points (for each case)

1. (6 points) Write out ∃!xP(x), where the domain consists of the integers 1, 2, and 3, in terms of negations, conjunctions, and disjunctions.

2 points + 2 points + 2points

1. (30 points) Let F(x, y) be the statement “x can fool y,” where the domain consists of all people in the world. Use quantifiers to express each of these statements.
   1. Everybody can fool Fred.
   2. Evelyn can fool everybody.
   3. Everybody can fool somebody.
   4. There is no one who can fool everybody.
   5. Everyone can be fooled by somebody.

3 points each (binary)

* 1. No one can fool both Fred and Jerry.
  2. Nancy can fool exactly two people.

**which is equivalent to**

* 1. There is exactly one person whom everybody can fool.

**which is equivalent to**

* 1. No one can fool himself or herself.

**which is equivalent to**

* 1. There is someone who can fool exactly one person besides himself or herself.

**which is equivalent to**

1. (4 points) Use predicates, quantifiers, logical connectives, and mathematical operators to express the statement that there is a positive integer that is not the sum of three squares.

**which is equivalent to**

1 + 1 + 1 + 1

1. (16 points) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
   1. ∀x(x ≠ 0 → ∃y(xy = 1)) **True** +2
   2. ∃x∀y(y ≠ 0 → xy = 1) **False** +2
   3. ∀x∃y(x + y = 1) **True** +3
   4. ∃x∃y(x + 2y = 2 ∧ 2x + 4y = 5) **False** +3
   5. ∀x∃y(x + y = 2 ∧ 2x − y = 1) **False** +3
   6. ∀x∀y∃z(z = (x + y)/2) **True** +3