King Fahd University of Petroleum and Minerals
Information and Computer Science Department

ICS 253: Discrete Structures I

Homework Assignment #5
(Due Friday July 12, 2013 at midnight)

Answer the following questions. NOTE: There are <u>10 extra points</u> in this assignment.

1. (10 points) Section 5.3: Recursive Definitions and Structural Induction
    a. Use structural induction to show that $l(T)$, the number of leaves of a full binary tree T, is 1 more than $i(T)$, the number of internal vertices of T.

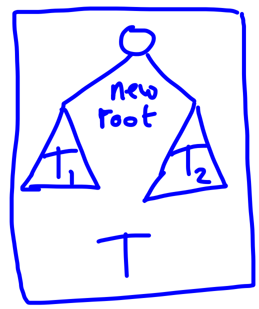
1. Basis: A full binary tree with 1 root node satisfies

$1 = l(T) = 1 + i(T)$ where $i(T) = 0$.

(0, 2)

2. Recursive step: Assume that the above result holds for T_1 & T_2.

The new root is an internal node



$\therefore i(T) = i(T_1) + i(T_2) + 1$ ..... (*)

Note that $l(T) = l(T_1) + l(T_2)$ ...... (#)

By induction hypothesis, T_1 & T_2 satisfy the rule, i.e. $i(T_1) + 1 = l(T_1)$, and $i(T_2) + 1 = l(T_2)$. .... (**)

$\therefore i(T) = l(T_1) - 1 + l(T_2) - 1 + 1$ from plugging (**) in (*)

$= l(T_1) + l(T_2) - 1$

$= l(T) - 1$ (from (#))

i.e. $l(T) = i(T) + 1$.

(0, 5, 8)

2. (66 points) Section 6.1: The Basics of Counting

a. (8 points) A multiple-choice test contains 20 questions. There are four possible answers for each question.

i. In how many ways can a student answer the questions on the test if the student answers every question?

4^{20}

(0, 4)

ii. In how many ways can a student answer the questions on the test if the student can leave answers blank?

5^{20}

(0, 4)

b. (4 points) How many bit strings are there of length six or less, not counting the empty string?

1-bit: 2, 2-bit: 2^2, 3-bit: 2^3, ...

$\therefore \text{Total} = \sum_{i=1}^{6} 2^i$

(0, 2, 4)

c. (12 points) How many strings of four decimal digits

i. do not contain the same digit twice?

$(10)(9)(8)(7)$

(0, 4)

ii. end with an even digit?

$10^3(5)$

(0, 4)

iii. have exactly three digits that are 9s?

X999, 9X99, 99X9, 999X with $X \in \{0, 1, \ldots, 7, 8\}$

$9 \times 4 = 36$.

(0, 4)

d. (16 points) How many strings of eight uppercase English letters are there

i. if letters can be repeated?

26^8 (0, 2)

ii. if no letter can be repeated?

$(26)(25)(24)(23)(22)(21)(20)(19)$ (0, 2)

iii. that start with X, if letters can be repeated?

26^7 (0, 2)

iv. that start with X, if no letter can be repeated?

$(25)(24)(23)(22)(21)(20)(19)$

(0, 2)

v. that start and end with X, if letters can be repeated?

26^6 (0, 2)

vi. that start with the letters BO (in that order), if letters can be repeated?

26^6 (0, 2)

vii. that start and end with the letters BO (in that order), if letters can be repeated?

26^4 (0, 2)

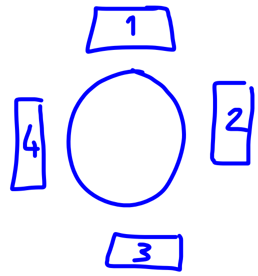
viii. that start or end with the letters BO (in that order), if letters can be repeated?

$26^6 + 26^6 - 26^4$ (0, 2)

e. (8 points) How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

$$\overbrace{\underbrace{(10 \times 9 \times 8 \times 7)}_{+4}/4}^{+4}$$

Since each seating corresponds to 4 different Assignments.



f. (10 points) The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character in the string must be a letter, either uppercase or lowercase, or an underscore. If the name of a variable is determined by its first eight characters, how many different variables can be named in C? (Note that the name of a variable may contain fewer than eight characters.)

# of characters	# of Variables
1	53
2	$53(63)$
3	$53(63)^2$
4	$53(63)^3$
5	$53(63)^4$
6	$53(63)^5$
7	$53(63)^6$
8	$53(63)^7$

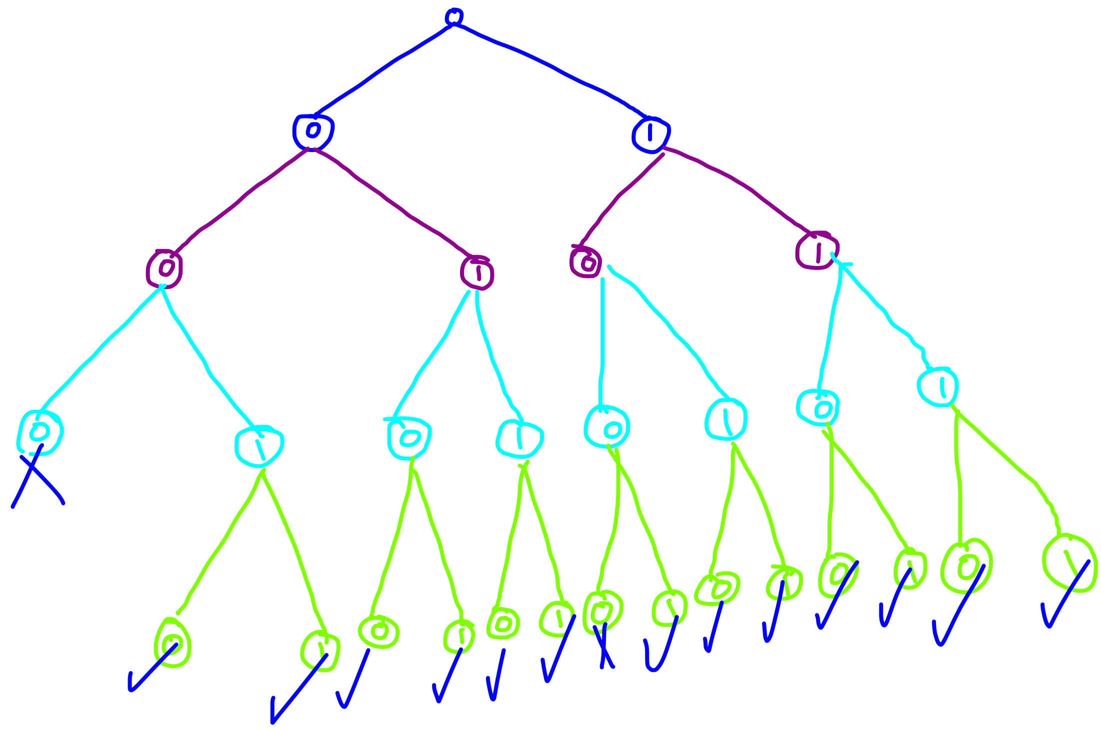
+6

$\therefore$ Total $= \underbrace{53\sum_{i=0}^{7}(63)^i}$

+4

or +10 (if above table is missing)

g. (8 points) Use a tree diagram to find the number of bit strings of length four with no three consecutive 0s.



(0,5,8)

∴ there are 13 of them.

3. (34 points) Section 6.2: The pigeonhole principle
   a. (8 points) A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
      i. How many balls must she select to be sure of having at least three balls of the same color?

The smallest "N" s.t. $\lceil \frac{N}{2} \rceil = 3$ i.e. $2 < \frac{N}{2} \leq 3$, iff $4 < N \leq 6$

$\therefore N = 5$

(0, 2, 4)

      ii. How many balls must she select to be sure of having at least three blue balls?

$10 + 3 = 13$ balls.

(0, 4)

b. (8 points) Suppose that there are nine students in a discrete mathematics class at a small college.

i. Show that the class must have at least five male students or at least five female students.

Since humans are either male or female, we have $\lceil \frac{9}{2} \rceil = 5$ either male or female

(0/4)

[or by contradiction like below]

ii. Show that the class must have at least three male students or at least seven female students.

if not, there would be at most 2 males & 6 females for a total of 8 *contradiction*

(0/4)

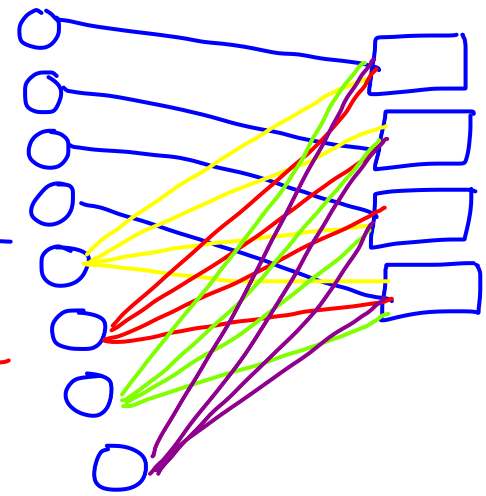
c. (8 points) Find an increasing subsequence of maximal length and a decreasing subsequence of maximal length in the sequence

22, 5, 7, 2, 23, 10, 15, 21, 3, 17.

5, 7, 10, 15, 21 (or any sequence of length 5) (0/4)

22, 10, 3 (or any sequence of length 3) (0/4)

d. (10 points) Find the least number of cables required to connect eight computers to four printers to guarantee that for every choice of four of the eight computers, these four computers can directly access four different printers. Justify your answer.



Connect the first 4 computers to the first four printers. Then connect the rest of the computers to all printers for a total number of $4+16=20$ cables.

(0.5)

To show that this is the minimum, assume that we can manage with 19 connections. Since we have 4 printers, at least 1 printer is connected to at most $\lfloor \frac{19}{4} \rfloor = 4$ computers (otherwise, the # of connections would be $5(4)=20$.)

(0.5)

$\therefore$ If this printer is not active, I still have 4 other computers that have access to only 3 printers.