**King Fahd University of Petroleum and Minerals**

**Information and Computer Science Department**

**ICS 253: Discrete Structures I**

**Homework Assignment #1 Possible Solutions**

**(Due Thursday June 13, 2013 at midnight)**

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1. (6 points) Which of these are propositions? What are the truth values of those that are propositions?
   1. (1 point) Do not pass go. **Not Proposition**
   2. (1 point) What time is it? **Not Proposition**

Binary for each answer

* 1. (1 point) There are no black flies in Maine. **False**
  2. (1 point) 4 + x = 5. **Not Proposition**
  3. (1 point) The moon is made of green cheese. **False**
  4. (1 point) 2*n* ≥ 100. **Not Proposition**

1. (6 points) Let p and q be the propositions “The election is decided” and “The votes have been counted,” respectively. Express each of these compound propositions as an English sentence.
   1. (2 points) p ∨ q **The election is decided or the votes have been counted**
   2. (2 points) q → p **If the votes have been counted, the election is decided**

Binary

* 1. (2 points) ¬q ∨ (¬p ∧ q)

**Either votes have not been counted, or they have been** **counted but the election is not decided.**

1. (12 points) Let *p*, *q*, and *r* be the propositions

*P*:You get an A on the final exam.

*Q* :You do every exercise in this book.

*R* :You get an A in this class.

Write these propositions using *p*, *q*, and *r* and logical connectives (including negations).

* 1. (3 points) You get an A in this class, but you do not do every exercise in this book.

**R ∧ ¬Q**

* 1. (3 points) You get an A on the final, but you don’t do every exercise in this book; nevertheless, you get an A in this class.

Binary

**P ∧ ¬Q ∧ R**

* 1. (3 points) Getting an A in this class follows from getting an A on the final.

**P → R**

* 1. (3 points) Unless you get an A on the final and do every exercise in this book, you will not get an A in this class.

**(¬P ∨** **¬Q) → ¬R**

1. (12 points) Write each of these propositions in the form “p if and only if q” in English.
   1. (3 points) For you to get an A in this course, it is necessary and sufficient that you learn how to solve discrete mathematics problems.

**You get an A in this course if and only if you learn how to solve discrete mathematics problems**

* 1. (3 points) If you read the newspaper every day, you will be informed, and conversely.

**You read the newspaper every day if and only if you will be informed**

Binary

* 1. (3 points) It rains if it is a weekend day, and it is a weekend day if it rains.

**It rains if and only if it is a weekend day**

* 1. (3 points) You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him.

**You can see the wizard if and only if he [the wizard] is not in.**

1. (6 points) State the converse, contrapositive, and inverse of the following statement: “When I stay up late, it is necessary that I sleep until noon.”

**Converse: If I sleep until noon, I stay up late**

Binary, 2 points each

**Contrapositive: If I don’t sleep until noon, I won’t stay up late**

**Inverse: If I don’t stay up late, I do not sleep until noon.**

1. (4 points) Construct a truth table for ((p ∧ q) ∨ ¬r) ↔ (r → (p ∧ q)). Specify whether it is a tautology, contingency or contradiction.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **¬r** | **p ∧ q** | **((p ∧ q)∨ ¬r)** | **(r → (p ∧ q))** | **((p ∧ q)∨ ¬r) ↔ (r → (p ∧ q))** |
| **F** | **F** | **F** | **T** | **F** | **T** | **T** | **T** |
| **F** | **F** | **T** | **F** | **F** | **F** | **F** | **T** |
| **F** | **T** | **F** | **T** | **F** | **T** | **T** | **T** |
| **F** | **T** | **T** | **F** | **F** | **F** | **F** | **T** |
| **T** | **F** | **F** | **T** | **F** | **T** | **T** | **T** |
| **T** | **F** | **T** | **F** | **F** | **F** | **F** | **T** |
| **T** | **T** | **F** | **T** | **T** | **T** | **T** | **T** |
| **T** | **T** | **T** | **F** | **T** | **T** | **T** | **T** |

-0.5 for each mistake in the table (up to 3.5) and 0.5 points for the “tautology”

**It is a tautology.**

1. (6 points) Are these system specifications consistent? “Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded.”

**p ≡ System software is being upgraded**

1.5 points

**q ≡ Users can access the file system**

**r ≡ Users can save new files**

**p → ¬q**

1.5 points

**q → r**

**¬r → ¬p**

2 points

**Since having r = true, p = false and q = true makes all implications above true,**

1 point

**the system is consistent.**

1. (10 points) The following exercises relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusions?
   1. (5 points) A says “The two of us are both knights” and B says “A is a knave.”

|  |  |  |  |
| --- | --- | --- | --- |
|  | **A Possibilities** | **B Possibilities** | **Joining the result with B statement** |
| **A is Knight** | **Knight** | **Knight** | **Not possible since B says A is knave.** |
| **A is Knave** | **Knight** | **Knave** | **Not Possible** |
| **Knave** | **Knight** | **Possible** |
| **Knave** | **Knave**  3 points for logic  2 points for right answer (binary) | **Not possible** |

**Therefore, A is knave and B is knight.**

* 1. (5 points) A says “We are both knaves” and B says nothing.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **A Possibilities** | **B Possibilities** | **Joining the result with B statement** |
| **A is Knight** | **Knave** | **Knave** | **Not possible since A cannot be knight and knave at the same time** |
| **A is Knave** | **Knave** | **Knight** | **Possible** |
|  | **Knight** | **Knave** | **Not possible since A is knave** |
|  | **Knight** | **Knight** | **Not possible since A is knave** |

**Therefore, A is knave and B is knight.**

3 points for logic

2 points for right answer (binary)

1. (8 points) Without using a truth table, show that

¬s → (r → ¬q) is equivalent to (r ∧ ¬s) → ¬q.

**¬s → (r → ¬q) ↔ s ∨ (r → ¬q)**

**↔ s ∨ (¬r ∨ ¬q)**

If there are mistakes, 4 points

**↔ (s ∨ ¬r) ∨ ¬q**

**↔ ¬ [(¬s ∧ r)] ∨ ¬q**

**↔ (¬s ∧ r) → ¬q**

**↔ (r ∧ ¬s) → ¬q**

1. (4 points) Show that (p ∧ q) → r and (p → r) ∧ (q → r) are **not** logically equivalent.

**If p = true and q = false and r = false, then**

Binary

**(p ∧ q) → r ↔ (T ∧ F) → F ↔ F → F ↔ T, and**

**(p → r) ∧ (q → r) ↔ (T → F) ∧ (F → F) ↔ F ∧ T ↔ F.**

1. (2 points) Let P(x) be the statement “the word x contains the letter a.” What are these truth values?
   1. (1 point) P(orange) **T**

Binary

* 1. (1 point) P(lemon) **F**

1. (12 points) Let N(x) be the statement “x has visited Abha,” where the domain consists of the students in your school. Express each of these quantifications in proper English.
   1. (2 points) ∃xN(x)

**Some [At least one] [One] student has visited Abha**

* 1. (2 points) ∀xN(x)

**All students have visited Abha**

Binary

* 1. (2 points) ¬∃xN(x)

**No student has visited Abha**

* 1. (2 points) ∃x¬N(x)

**There is a student who has not visited Abha**

* 1. (2 points) ¬∀xN(x)

**There is a student who has not visited Abha**

* 1. (2 points) ∀x¬N(x)

**No student has visited Abha**

1. (12 points) Let C(x) be the statement “x has a cat,” let L(x) be the statement “x has a lovebird,” and let F(x) be the statement “x has fish.” Express each of these statements in terms of C(x), L(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.
   1. (2 point) A student in your class has a cat, a lovebird, and fish.

**∃x (C(x) ∧ L(x) ∧ F(x))**

* 1. (2 points) Some student in your class has a lovebird and fish, but not a cat.

**∃x (L(x) ∧ F(x) ∧ ¬C(x))**

Binary

* 1. (2 points) All students in your class have a cat, a lovebird, or fish.

**∀x (C(x) ∨ L(x) ∨ F(x))**

* 1. (2 points) No student in your class has a cat and fish.

**¬∃x (C(x) ∧ F(x)) ≡ ∀x (¬C(x) ∨ ¬F(x))**

* 1. (4 points) For each of the three animals, cats, lovebirds, and fish, there is a student in your class who has this animal as a pet.

**∃x C(x) ∧ ∃y L(y) ∧ ∃z F(z)**

**IMPORTANT NOTE REGARDING YOUR SUBMISSION.**

1. Your homework submission must be a pdf file. If you do not type your homework and just solve by hand, make sure you clearly scan the pages of your homework and convert it to **A SINGLE pdf file**.
2. Please submit a pdf file with the following filename:

***Name\_ID\_Assignment1.pdf***

For example: If your name is ***Wasfi Ghassan Al-Khatib*** and your ID is ***201043210***, your filename should be

***WasfiAlKhatib\_201043210\_Assignment1.pdf***