King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 253-01: Discrete Structures I
Summer 2012-2013
Quiz#4, Sunday July 7, 2013.

Name:

ID#:

1. (10 points) Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set
   a. All bit strings not containing the bit 0.
   b. The real numbers containing a finite number of 1's in their decimal representation.
   c. The set $A \times Z^+$ where $A = \{2, 3\}$

a. Countably infinite. 2

2 $f(n) =$ the n-bit string with all 1's

2 b. Uncountable.

2 c. Countably infinite

$$f(n) = \begin{cases} (2, \frac{n}{2}) & n \text{ is even} \\ (3, \frac{n+1}{2}) & n \text{ is odd} \end{cases}$$ 2

2. (10 points) Let $P(n)$ be the statement that $1+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{n^2}<2-\frac{1}{n}$ where n is an integer greater than 1.
   a. (1 point) What is the statement $P(2)$?
   b. (1 point) Show that $P(2)$ is true, completing the basis step of the proof.
   c. (2 points) What is the inductive hypothesis?
   d. (1 point) What do you need to prove in the inductive step?
   e. (5 points) Complete the inductive step.

1 (a) $1+\frac{1}{4}<2-\frac{1}{2}$

1 (b) $1\frac{1}{4}<1\frac{1}{2}$ which is true

2 (c) Assume $P(n)$ holds, i.e.

$$1+\frac{1}{2^2}+\cdots+\frac{1}{n^2}<2-\frac{1}{n}$$

1 (d) To show that

$$1+\frac{1}{2^2}+\cdots+\frac{1}{(n+1)^2}<2-\frac{1}{n+1}$$

2 (e)

$$1+\frac{1}{2^2}+\cdots+\frac{1}{(n+1)^2}=1+\frac{1}{2^2}+\cdots+\frac{1}{n^2}+\frac{1}{(n+1)^2}$$

$$<2-\frac{1}{n}+\frac{1}{(n+1)^2}$$

2

$$=2+\frac{1}{(n+1)^2}-\frac{1}{n}$$

$$=2+\frac{n-(n+1)^2}{n(n+1)^2}$$

$$=2+\frac{n-n^2-2n-1}{n(n+1)^2}$$

$$=2-\frac{n^2+n+1}{n(n+1)^2}$$

1

$$=2-\frac{n^2+n}{n(n+1)^2}-\frac{1}{n(n+1)^2}$$

$$=2-\frac{1}{n+1}-\frac{1}{n(n+1)^2}$$

$$\leq 2-\frac{1}{n+1}\quad \left(\text{since } \frac{1}{n(n+1)^2}\geq 0\right)$$