King Fahd University of Petroleum and Minerals
College of Computer Science and Engineering
Information and Computer Science Department

ICS 253-01: Discrete Structures I
Summer Session 2016-2017
Quiz#3, Thursday July 27, 2017.

Name:

ID#:

1. (10 points) Show that $3x + 2$ is even if and only if $x + 5$ is odd.

$\Rightarrow$ Assume that $3x+2$ is even.

Then $\exists i \in \mathbb{Z}$ s.t. $3x+2 = 2i$

$3x = 2i - 2$

$x = -2x + 2i - 2$

$= 2(-x + i - 1)$

but $-x + i - 1 \in \mathbb{Z} \qquad \in \mathbb{Z}$.

$\therefore$ x is even.

$\therefore$ $x + 5 = x + 4 + 1 = 2(-x + i - 1 + 2) + 1$

is obviously odd

$\Leftarrow$ if $x + 5$ is odd, then

$x + 5 = 2j + 1 \qquad j \in \mathbb{Z}$.

$x = 2j - 5 + 1 = 2j - 4$

$= 2(j - 2)$

Since $j - 2 \in \mathbb{Z}$,

$x = 2(j - 2)$ is even.

$3x = 2[3(j-2)]$ is even

$3x + 2 = 2[3(j-2)+1]$ is even

2. (3 points) Determine whether each of these statements is true or false.

   a) $x \in \{x\}$ T

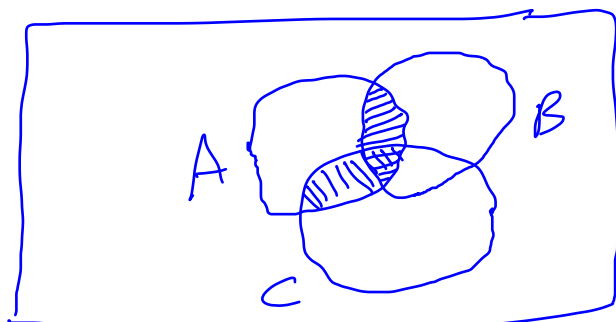   **b)** $\{x\} \subseteq \{x\}$ T

   c) $\{x\} \in \{x\}$ F

   d) $\{x\} \in \{\{x\}\}$ T

   e) $\emptyset \subseteq \{x\}$ T

   f) $\emptyset \in \{x\}$ F

3. (3 points) Draw the Venn diagram for $(A \cap B) \cup (A \cap C)$ of the sets A, B, and C.

4. (4 points) Find $\bigcap_{i=1}^{\infty} A_i$ and $\bigcup_{i=1}^{\infty} A_i$ where $A_i = [-i, i]$ (the closed interval).

$$\bigcap_{i=1}^{\infty} A_i = \bigcap_{i=1}^{\infty} [-i, i] = [-1, 1] \cap [-2, 2] \cap \cdots$$

$$= [-1, 1]$$

$$\bigcup_{i=1}^{\infty} A_i = [-1, 1] \cup [-2, 2] \cup \cdots$$

$$= \mathbb{R}$$