King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 253-01: Discrete Structures I
Summer Session 2016-2017
Quiz#2, Wednesday July 19, 2017.

Name:

ID#:

1. (10 points) Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. Assume that the universe of discourse is all people in the world. Make sure that you first you define your propositional functions.

$p(x) \equiv$ x is perfect
$q(x) \equiv$ x is your friend

   a. No one is perfect.

   $\forall x \neg p(x) \Leftrightarrow \neg \exists x\, p(x)$

   b. Not everyone is perfect.

   $\neg \forall x\, p(x) \Leftrightarrow \exists x \neg p(x)$.

   c. All your friends are perfect.

   $\forall x (q(x) \rightarrow p(x))$

   d. At least one of your friends is perfect.

   $\exists x (q(x) \wedge p(x))$

2. (4 points) Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1, 2 or 3 and y is 2 or 3. Write out the proposition $\forall y \exists x P(x, y)$ using disjunctions and conjunctions, only.

$(p(1,2) \vee p(2,2) \vee p(3,2)) \wedge$
$(p(1,3) \vee p(2,3) \vee p(3,3))$

3. (6 points) Show that $(\exists x P(x)) \wedge A \equiv \exists x(P(x) \wedge A)$ where x does not occur as a free variable in A. Assume that the domain is nonempty.

Since x doesn't appear in A as a free variable we have 2 cases for A.

1. $A =$ True.

In this case, the left hand side (LHS) becomes $(\exists x\, P(x)) \wedge T \equiv \exists x\, P(x)$ .... ①

& the right hand side (RHS) becomes

$\exists x(P(x) \wedge T) \equiv \exists x\, P(x)$. - - ②

① & ② are equal.

2. $A =$ False.

In this case, the LHS becomes

$(\exists x\, P(x)) \wedge F \equiv F$ . . . . ③

The RHS is $\exists x(P(x) \wedge F)$

$\Leftrightarrow \exists x(F)$ - - ④

$\Leftrightarrow F$

So, ③ & ④ are equal.

$\therefore$ the equivalence holds.