

King Fahd University of Petroleum and Minerals
 College of Computer Science and Engineering
 Information and Computer Science Department

ICS 253-01: Discrete Structures I
 Summer Session 2016-2017
 Quiz#5, Thursday August 10, 2017.

Name:

ID#:

1. (10 points) Let $P(n)$ be the statement that a postage of n cents can be formed using just 3-cent stamps and 5-cent stamps. A proof by strong induction can be developed.

a. Show that the statements $P(8)$, $P(9)$, and $P(10)$ are true.

$$P(8): 1(3) + 1(5) = 8.$$

$$P(9): 3(3) + 0(5) = 9.$$

$$P(10): 0(3) + 2(5) = 10.$$

b. What is the inductive hypothesis of the proof?

Assume that values k where $10 \leq k \leq n$ can be formed using 3-cent & 5-cent stamps.

c. What do you need to prove in the inductive step?

To show that 3-cent & 5-cent stamps can form a value of $n+1$

d. Complete the inductive step for $k \geq 10$.

To form value $n+1$, use the inductive hypothesis to find i & $j \in \mathbb{Z}$ s.t. $3i + 5j = n-2$.
 Adding a 3-cent stamp, we get
 $3i + 3 + 5j = 3(i+1) + 5j = n+1$.
 \therefore use $i+1$ 3-cent stamps & j 5-cent stamps.

2. (4 points) Give a recursive definition of the sequence $\{a_n\}, n = 1, 2, 3, \dots$ if $a_n = n^2$.

$$a_1 = 1$$

$$a_{n+1} = (n+1)^2 = n^2 + 2n + 1 = a_n + 2n + 1$$

∞
 ∞

$$a_{n+1} = a_n + 2n + 1 \quad n > 1$$

$$= 1 \quad n = 1$$

3. (6 points) Define well-formed formulae of sets, variables representing sets, and operators from $\{\bar{}, \cup, \cap, -\}$

① \emptyset, U and the variable A constitute well-formed formulae.

② if A & B are ~~set variables~~ well-formed formulae, then

(\bar{A}) is a well-formed formulae S

$(A \cap B)$ S S S S

$(A \cup B)$ S S S S

$(A - B)$ S S S S