King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 253-01: Discrete Structures I
Summer 2016-2017
Quiz#4, Wednesday August 2, 2017.

Name:

ID#:

1. (6 points) Suppose that g is a function from A to B and f is a function from B to C. Show that if both f and g are one to one function, then $f \circ g$ is one to one.

f is 1:1 $\rightarrow$ $f(b_1) = f(b_2) \leftrightarrow b_1 = b_2$.

g is 1:1 $\rightarrow$ $g(a_1) = g(a_2) \leftrightarrow a_1 = a_2$.

To show that if $(f \circ g)(x) = (f \circ g)(y)$ $(x, y \in A)$ then $x = y$.

Let $(f \circ g)(x) = (f \circ g)(y)$.

Then $f(g(x)) = f(g(y))$.

but f is 1:1, Hence, $g(x) = g(y)$.

In addition, g is 1:1.

$\therefore x = y$.

$\therefore f \circ g$ is 1:1.

2. (6 points) Find

$$\sum_{k100}^{400} k$$

given that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$$\sum_{k100}^{400} k = \sum_{k=1}^{400} k - \sum_{k=1}^{99} k$$

$$= \frac{(400)(401)}{2} - \frac{99(100)}{2}.$$

3. (8 points) Is the sequence $a_n = (-4)^n$ a solution to the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$? Justify your answer.

Substituting a_{n-1} & a_{n-2} in the recurrence, we get

$$8(-4)^{n-1} - 16(-4)^{n-2}$$

$$= (-2)(-4)(-4)^{n-1} - (-4)(-4)(-4)^{n-2}$$

$$= -2(-4)^n - (-4)^n$$

$$= -3(-4)^n \neq (-4)^n .$$

∴ It is not a solution to the recurrence.

OR

From $a_n = (-4)^n$, $a_0 = 1, a_1 = -4, a_2 = 16$

but $16 \neq 8(-4) - 16(1)$

$= -32 - 16 = -48$

∴ a_n is not a solution