

# King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering  
Information and Computer Science Department

ICS 353-03: Design and Analysis of Algorithms

Spring 2006-2007

Quiz#3, Monday April 2, 2007.

Name:

ID#:

1. (12 points) MinMax Algorithm:

- (8 points) Write down the divide and conquer algorithm that returns the minimum and the maximum of an array.
- (4 points) List the three main components of a divide and conquer algorithm and identify them in the MinMax algorithm above.

Algorithm MinMax (A, low, high)

1.5 — 1. if  $high - low == 1$  then  
if  $A[low] \leq A[high]$   
return (A[low], A[high])  
else  
return (A[high], A[low]);

1.5 — 2.  $Mid = (high - low) / 2$ ;

1.5 — 3.  $(x_1, y_1) = \text{MinMax}(A, low, Mid)$ ;

1.5 — 4.  $(x_2, y_2) = \text{MinMax}(A, Mid+1, high)$ ;

2 — 5. if  $x_1 \leq x_2$  then  
 $x = x_1$   
else  $x = x_2$

6. if  $y_1 > y_2$  then  
 $y = y_1$   
else  $y = y_2$

7. return (x, y);

- ① The three main components are Divide, Conquer & combine.
- ① Line 2 : Divide
- ② Lines 3 & 4 : Conquer
- ③ Lines 5 & 6 : Combine.

2. (8 points) Find the solution of the following recurrence equation in  $\Theta()$  notation:

$$f(n) = \begin{cases} 1 & n < 4 \\ 2f(n/3) + \sqrt{n} & n \geq 4 \end{cases}$$

+2  $\alpha=2$   $b=3$   $n^{\log_a b} = n^{\log_3 2}$

+2  $g(n) = n^{1/2} = O(n^{\log_3 2 - \epsilon})$  where  $0 < \epsilon < 0.01$ .

[Note that  $\log_3 2 = \frac{\log_2 2}{\log_2 3} = \frac{1}{\log_2 3} > \frac{1}{\log_2 4} = \frac{1}{2}$ ]

+2  $\therefore$  Case 1 of the Master theorem applies.

+2  $\therefore f(n) = \Theta(n^{\log_3 2})$