

# King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering  
Information and Computer Science Department

ICS 353-04: Design and Analysis of Algorithms

Spring 2006-2007

Quiz#5, Monday May 28, 2007.

Name:

ID#:

## 1. (14 points) NP-Complete Problems

- (4 points) Explain clearly how one can prove that a problem is NP-Complete using the transitivity property of polynomial time reductions.
- (10 points) Consider the following instance of Satisfiability:

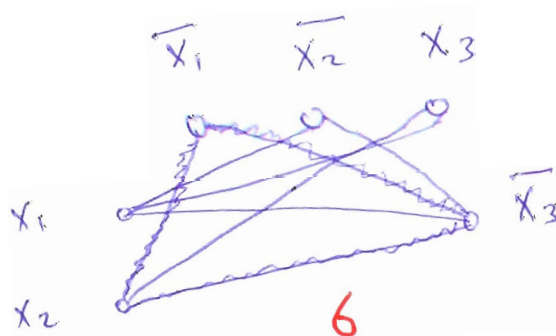
$$(\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_3}) \wedge (x_1 \vee x_2)$$

- (6 points) Following the reduction method from satisfiability to clique, transform the above formula into an instance of clique for which the answer is yes if and only if the above formula is satisfiable.
  - (4 points) Find a clique of size 3 in your graph and convert it into a satisfying assignment for the formula given above.
2. (6 points) Explain, in general terms, how can backtracking assist in solving hard problems.

1. a. Transitivity is used to show that a problem  $\pi \in \text{NP-hard}$ .  
Finding a reduction from a known NP-Complete problem,  $\pi'$  to  $\pi$ ,  $\pi' \leq_{\text{poly}} \pi$  implies that using transitivity,

$\forall \pi \in \text{NP}, \pi \leq_{\text{poly}} \pi' \text{ \& \; since } \pi' \leq_{\text{poly}} \pi \Rightarrow \forall \pi \in \text{NP}, \pi \leq_{\text{poly}} \pi$   
 $\downarrow$   $\downarrow$   $\uparrow$   
 $\pi' \in \text{NP-complete}$   $\text{known}$   $\pi \in \text{NP-hard}$

1. b. i.



ii.  $\overline{x_1} = \text{True} \Leftrightarrow x_1 = \text{False}$   
 $\overline{x_3} = \text{True} \Leftrightarrow x_3 = \text{False}$   
 $x_2 = \text{True}$   
(F, T, F) or (T, F, F) 2+2

2. Backtracking reduces the search space by terminating the inspection of many "possible" answers that are "wrong" answers.

$$3 + 3$$