

King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 353-04: Design and Analysis of Algorithms

Spring 2006-2007

Quiz#2, Monday March 12, 2007.

Name:

ID#:

1. (10 points) Consider the following algorithm:

```
1. count := 0;
2. for i := 1 to n do
3.   j := 2;
4.   while j <= n do
5.     j := j2;
6.     count := count + 1;
7.   end while
8. end for;
9. return count;
```

- a. (8 points) Evaluate the variable count by finding the number of times Line 6 gets executed.
- b. (2 points) Express the number of times Line 6 gets executed in terms of $\Theta()$ notation

$$\text{a. } \sum_{i=1}^n \sum_{k=0}^r 1$$

$$= \sum_{i=1}^n (r+1)$$

$$= (\log \log n + 1) \sum_{i=1}^n 1$$

$$= n \log \log n + n$$

b. $\Theta(n \log \log n)$

$$j = 2, 2^2, 2^{2^2}, 2^{2^3}, \dots, 2^{2^r}$$

where $n = 2^{2^r} \Leftrightarrow r = \log \log n$

$$k = 0, 1, 2, \dots, r$$

2. (10 points) Express the function $n^2 + 2^n$ in terms of $\Theta()$ notation, showing your work (i.e. a final answer without a proof is worth zero points).

+5 — $\lim_{n \rightarrow \infty} \frac{n^2}{2^n} = \lim_{n \rightarrow \infty} \frac{2n}{2^n \ln 2}$ (L'Hopital's Rule)

+1 — $= c_1 \lim_{n \rightarrow \infty} \frac{n}{2^n}$ (c_1 is constant)
 $= \frac{2}{\ln 2}$

+2 — $= c_1 \lim_{n \rightarrow \infty} \frac{1}{2^n \ln 2} = 0$

$\lim_{n \rightarrow \infty} n^2 = o(2^n)$

$\lim_{n \rightarrow \infty} n^2 + 2^n = \Theta(2^n)$

+1 —