

King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 353-03: Design and Analysis of Algorithms

Spring 2006-2007

Quiz#2, Monday March 12, 2007.

Name:

ID#:

1. (10 points) Express the function $\log^2 n + n^{1/2} + \log \log n$ in terms of $\Theta()$ notation, showing your work (i.e. a final answer without a proof is worth zero points).

+5

$$1. \lim_{n \rightarrow \infty} \frac{\log \log n}{\log^2 n} = \lim_{n \rightarrow \infty} \frac{\log \frac{\ln n}{\ln 2}}{\left(\frac{\ln n}{\ln 2}\right)^2}$$

$$= \lim_{n \rightarrow \infty} \frac{\ln \ln n - \ln \ln 2}{\left(\frac{1}{\ln 2}\right)^2 \ln^2 n}$$

$$= \lim_{n \rightarrow \infty} \frac{\ln \ln n}{\ln^2 n}$$

$$= c_1 \lim_{n \rightarrow \infty} \frac{\frac{1}{\ln n} \cdot \frac{1}{n}}{2 \ln n \cdot \frac{1}{n}}$$

$$= c_1 \lim_{n \rightarrow \infty} \frac{1}{2 \ln^2 n} = 0 \dots (*)$$

+4

$$2. \lim_{n \rightarrow \infty} \frac{\log^2 n}{n^{1/2}} = c \lim_{n \rightarrow \infty} \frac{\ln^2 n}{n^{1/2}}$$

$$= c_2 \lim_{n \rightarrow \infty} \frac{(\ln n)^{1/n}}{n^{1/2}}$$

$$= c_2 \lim_{n \rightarrow \infty} \frac{\ln n}{n^{1/2}} = \lim_{n \rightarrow \infty} \frac{1}{n^{1/2}} = 0 \dots (**)$$

+1

from $(*)$ & $(**)$ $f(n) = \Theta(n^{1/2})$

2. (10 points) Consider the following recursive algorithm:

```

Algorithm DoSomething(A, i, j)
1.  if (i >= j)
2.      return (i-j);
3.  else {
4.      A[j] = DoSomething(A, i, j-1); // i=1, j-1=n-1
5.      for (k=i; k < j; k++) // i=1, j=n
6.          if (A[k] < A[j])
7.              A[k] += A[j];
8.          else
9.              A[k] -= A[j];
10. }
    
```

Let $T(n)$ denote the number of *element comparisons* carried out by Algorithm DoSomething. Answer the following questions:

- (2 points) What is the value of $T(1)$?
- (8 points) Derive the recurrence equation describing the value of $T(n)$ for $n > 1$. (No need to solve the recurrence equation)

~~element~~ element comparisons occur in line 6, in addition to the recursive calls.

+2 a. $T(1) = 0$ (line 6 is not executed)

b. $T(n)$ Assume $i = 1, j = n$. ~~+2~~ +2

Then

$$T(n) = \underset{\substack{\uparrow \\ +2}}{T(n-1)} + \underset{\substack{\uparrow \\ +3}}{(n-1)}$$