King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 353-02: Design and Analysis of Algorithms
Fall Semester 2018-2019
Quiz#1, Tuesday September 11th, 2018.

Name:

ID#:

1. (10 points) Using the definition of Big Ω(), show that $n^2 - 7n + 1$ is in $\Omega(n^2)$.
To find $c > 0$ and $n_0 \in \mathbb{Z}^+$ such that

$$n^2 - 7n + 1 \geq cn^2 \quad \forall n \geq n_0$$

Let $c = \frac{1}{2}$. We solve the inequality to find n_0 as follows:

$$n^2 - 7n + 1 \geq \frac{n^2}{2} \Leftrightarrow \frac{n^2}{2} - 7n + 1 \geq 0$$
$$\Leftrightarrow n^2 - 14n + 2 \geq 0$$

Solving the equation $n^2 - 14n + 2 = 0$, we get

$$n = \frac{14 \mp \sqrt{(-14)^2 - 4(1)(2)}}{2} = 7 \mp \frac{\sqrt{188}}{2}$$
$$= 7 \mp \sqrt{47}$$

Hence, for $c = \frac{1}{2}$, we can choose $n_0 = 14$ such that $n^2 - 7n + 1 \geq \frac{n^2}{2} \quad \forall n \geq 14$.

2. (6 points) Let A [1..40] = 13, 14, 15, …, 51, 52. How many comparisons are performed by the binary search algorithm to search for
   a. (3 points) the value 22
   b. (3 points) the value 12

a.

$x = 22$

# comp.	low	high	mid	A[mid]
1	1	40	20	32
1	1	19	10	22

element comparisons = 2.

b.

$x = 12$

# comp.	low	high	mid	A[mid]
1	1	40	20	32
1	1	19	10	22
1	1	9	5	17
1	1	4	2	14
1	1	1	1	13
	1	0		

element comparisons = 5.

3. (4 points) With respect to the Merge Algorithm that merges two sorted arrays of lengths n_1 and n_2, respectively, what are the minimum number and maximum number of element comparisons? What are the minimum number and maximum number of element assignments?

	Minimum	Maximum
Element Comparisons	$\min\{n_1, n_2\}$	$n_1 + n_2 - 1$
Element Assignments	$2(n_1 + n_2)$	$2(n_1 + n_2)$