King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 353-02: Design and Analysis of Algorithms
Fall Semester 2018-2019
Quiz#2, Sunday September 30th, 2018.

Name:

ID#:

1. (10 points) Express the function $f(n) = n^2 + 2^n$ in terms of Big $\Theta()$ notation. Prove your answer.

$$\lim_{n\to\infty} \frac{n^2}{2^n} = \lim_{n\to\infty} \frac{n^2}{e^{\ln 2^n}}$$

$$= \lim_{n\to\infty} \frac{n^2}{e^{n\ln 2}}$$

$$= \lim_{n\to\infty} \frac{2n}{e^{n\ln 2}(\ln 2)} \quad (\text{L'Hospital's Rule})$$

$$= c_1 \lim_{n\to\infty} \frac{n}{e^{n\ln 2}}, \text{ where } c_1 = \frac{2}{\ln 2}$$

$$= c_1 \lim_{n\to\infty} \frac{1}{e^{n\ln 2}(\ln 2)}$$

$$= \frac{c_1}{\ln 2} \lim_{n\to\infty} \frac{1}{e^{n\ln 2}} = 0$$

So $f(n) = \Theta(2^n)$.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1} \qquad \sum_{i=1}^{n} \left(\frac{1}{2}\right)^i . i = 2 - \frac{n+2}{2^n} \qquad 2^{\lg n} = n$$

,

$$log_b a = \frac{log_c a}{log_c b} \text{ where } c, b \neq 1 \qquad \log a^b = b \log a \qquad \log ab = \log a + \log$$

2. (10 points) Consider the following algorithm:

```
1. sum = 0;
2. for (i=1; i <= n; i++)
3.   for (j=2; j <= (2*i); j+=2)
4.      sum++; // MyStatement
```

a. (4 points) Express the number of times step 4 gets executed in summation form.
b. (4 points) Evaluate the summation of part (a).
c. (2 points) Express the time complexity of the algorithm using Big Θ() notation.

a. The values of j are

$j: 2, 4, 6, 8, \ldots, 2i$

$j: 2(1), 2(2), 2(3), 2(4), \ldots, 2(i)$

Let $k = 1, 2, 3, \ldots, i$

where $j = 2k$. Then,

Step 4 is executed $\displaystyle\sum_{i=1}^{n} \sum_{k=1}^{i} 1$

b. $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

c. $\Theta(n^2)$.